Parameter Estimation for Tear Film Breakup Dynamics

Rayanne A. Luke[1], Richard J. Braun[1], Tobin A. Driscoll[1], Deborah Antwi[2], Carolyn G. Begley[2], P. Ewen King-Smith[3]

[1]Department of Mathematical Sciences, University of Delaware; [2]School of Optometry, Indiana University; [3]College of Optometry, The Ohio State University

INTRODUCTION

Tear film breakup (TBU) is often evaporation-driven [1]. Many parameters affect tear film (TF) thickness and fluorescent (FL) intensity distributions over time; exact values or ranges for some are not well known. We determine experimental quantities that cannot currently be measured *in vivo* during TBU via fitting to our models.

METHODS

- Separate procedures are used to estimate initial FL concentration and localized initial TF thickness (not shown) [2].
- A math model is fit to FL intensity data from normal subjects' TFs extracted from a spot or streak TBU via an optimization in Matlab®.
- The fit is optimized over:
  - $v_{\max}$, peak evaporation rate
  - $v_{\min}$, background evaporation rate
  - r_w, width of Gaussian evaporation profile
  - t_{on}, when non-uniform thinning is fully on

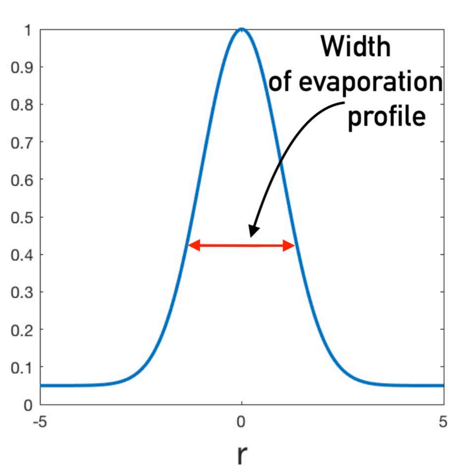

Figure 1: We adjust the width and the "fully on time" of evaporation

CONCLUSIONS

We **estimate tear film thickness** from fluorescent intensity images,

determine TBU thinning rates by fitting TBU with a math model,

and produce **results consistent with** experimental point **measurements.**

NEXT STEPS

- The fits are able to capture a large range of expected thinning rates.
- Thinning rates cannot be measured *in vivo* during TBU; peak and background rates fall within accepted experimental ranges of point measurements [3], [4].
- Next we will incorporate a model for rapid lipid-driven thinning and move to 2D models.

DETAILS OF THE MODEL

For Figure 3, we model h, TF thickness, and p, fluid pressure:

$$\frac{\partial h}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{h^3}{6}\frac{\partial p}{\partial r}\right) + Q_w, \qquad 0 < r < R_0$$

$$p = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right), \qquad Q_w = P_c(c-1) - J_e$$

where J_e is a Gaussian evaporation profile (see Fig. 1), and with constant P_c. We have transport equations for FL concentration f, and osmolarity c. We then compute FL intensity I from:

$$I = I_0 \frac{1-\exp(-\phi h f)}{1+f^2}.$$

We then solve

$$\underset{\{v_{\max},\, v_{\min},\, r_w,\, t_{on}\}}{\arg\min} \left\| I_{th}(r,t) - I_{ex}(r,t) \right\|_2^2$$

Note: For a flat film, $p = 0$ (see Figure 4).

RESULTS

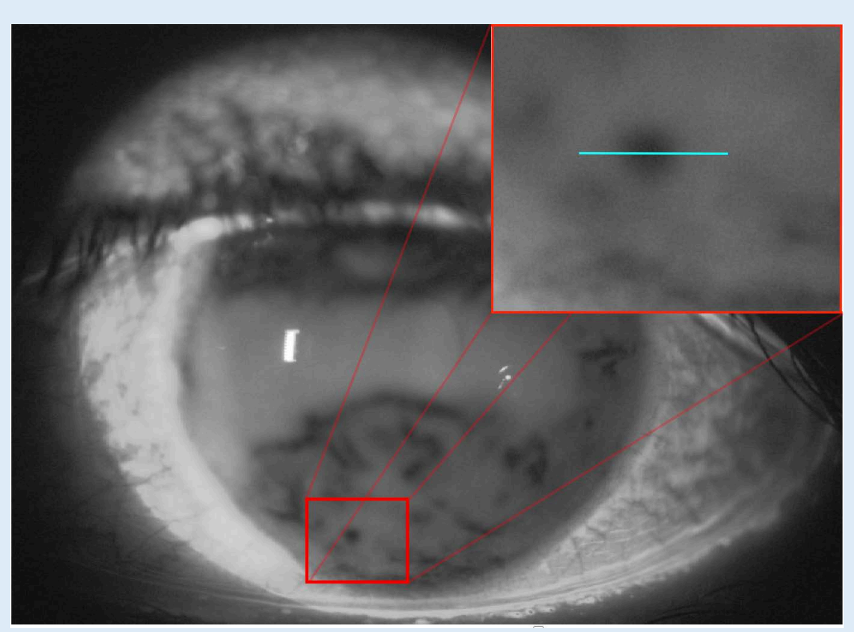

Figure 2: We extract a line of data from across a spot TBU. This is fit with our theoretical FL intensity function.

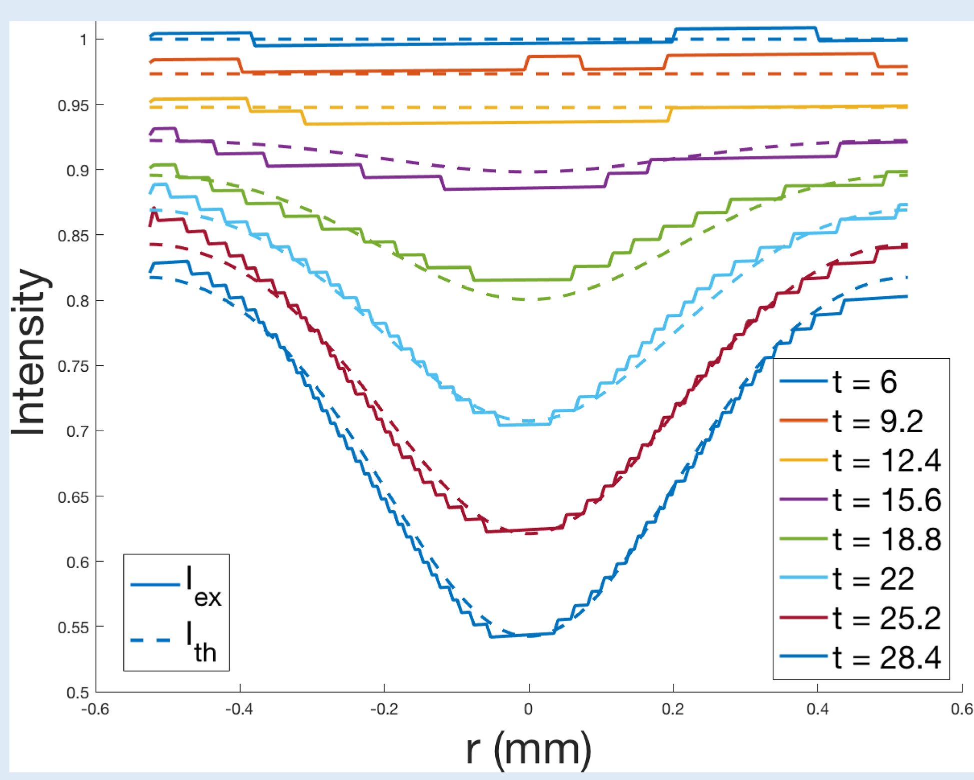

Figure 3: Spot TBU is fit with optimal values $v_{\max}$ = 7.80 μm/min, $v_{\min}$ = 1.92 μm/min, r_w = 0.727, and t_{on} = 10.9 s.

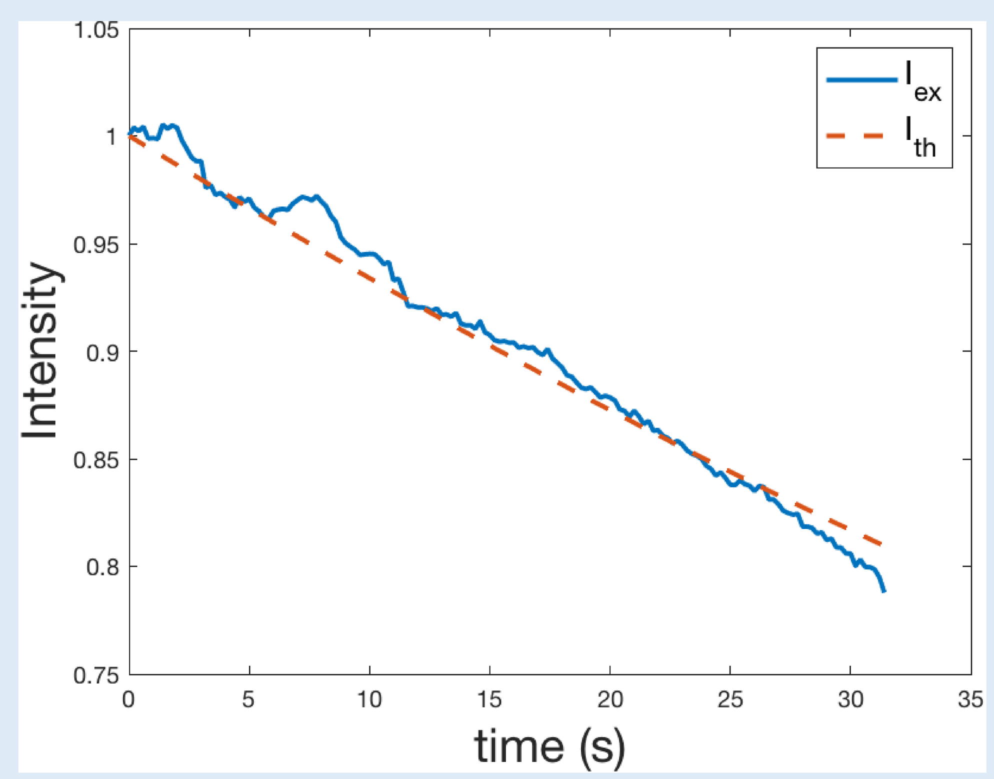

Figure 4: A flat region is fit with a constant optimal value v = 2.40 μm/min.

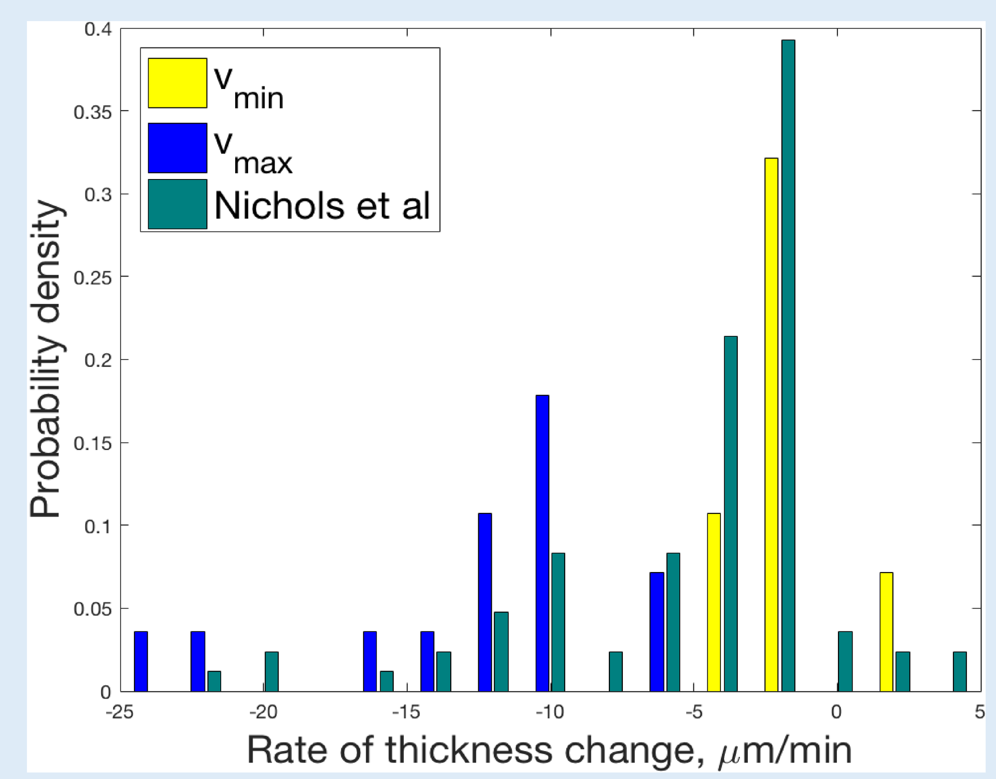

Figure 5: Histogram of thinning rates from optimized fits plotted alongside data from [4]. Our values are consistent with the distribution of point measurements reported in [4].

rayanne@udel.edu

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