A single equation model for Tear Breakup (TBU)

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INTRODUCTION

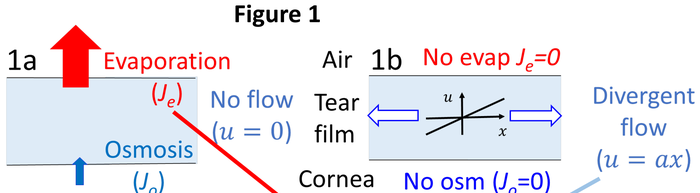
Simplified mathematical models for TBU dynamics and its fluorescent (FL) imaging are developed. This new ODE model captures some essential dynamics of some common types of TBU [1,2,3]. Etiologies of TBU include:

1. Type I: evaporation driven (Figure 1a) [1,4,5]
2. Type: II divergent flow driven (Figure 1b) with strength a
3. Type III: a mix of type I and type II.

METHOD

The single ODE for thickness $h(t)$ below is solved numerically in Matlab. With $h(t)$, we can compute osmolarity $c(t)$, fluorescein concentration $f(t)$, and intensity $I(t)$.

Figure 1



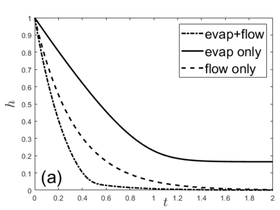
$$\frac{dh}{dt} = P_c\left(\frac{e^{-at}}{h} - 1\right) - 1 - ah, \qquad h(0) = 1$$

RESULTS

Type I: c and f ↗ as h ↘; osmosis stops thinning [1]. I is constant if f is dilute, I ↘ if self-quenching. [4,5,6]

Type II: (New) h ↘ from flow only; c and f *remain constant*. I ↘ from thinning due to flow only. [4,6]

Type III: (New) Similar to type I early on but like type II later.



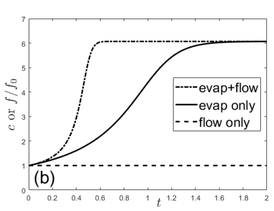
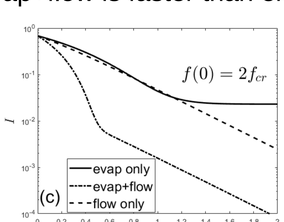


Figure 2: $J_e = 1$ or 0, $a = 1$ or 0.

(a) $h(t)$. **(b)** $c(t)$ or $f(t)/f_0$.

(c) $I(t)$ with initial f as twice the critical concentration f_{cr}.

Evap+flow is faster than either alone.



Conclusions:

A single equation can capture essential features of TBU from evaporation or divergent flow

This model is useful for education and for finding tear film and TBU parameters.

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References

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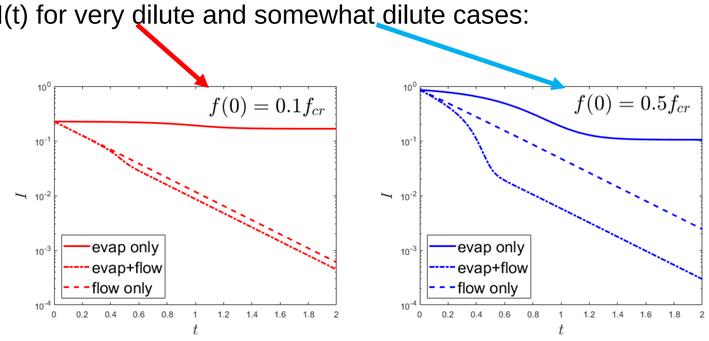
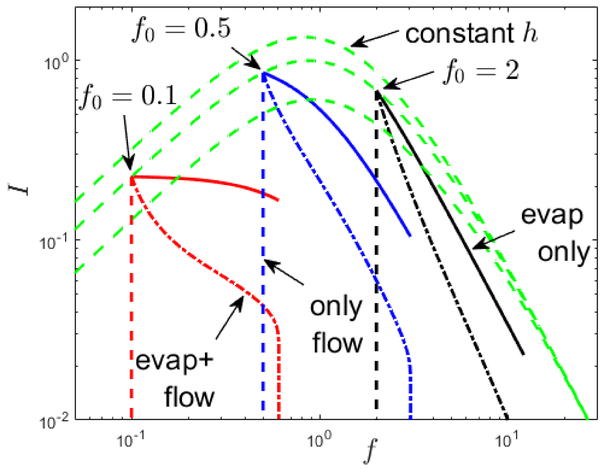
BONUS: OPTIONAL DETAILS

I(t) for very dilute and somewhat dilute cases:



Note how $I(t)$ changes much less for evaporative thinning! [4,6]

Summarizing results by plotting I *vs* f, as in [6]:



Notes:

(i) The divergent flow's cause is not specified; it could be from dewetting [2] or lipid spreading [3]. The simple flow here is extensional (an idealization).

(ii) The variables have been normalized. J_e is normalized with the thinning rate. h is normalized with initial thickness (3.5 μm). c is normalized with the isotonic values (300 mOsM). f is normalized with the critical fluorescein concentration ($f_{cr} = 0.2\%$) [4,5].

(iii) The intensity is computed as in [4] and subsequent papers [1,6,7]: $I = I_0\frac{1-e^{-\varphi hf}}{1+f^2}$ (φ, I_0 are constants)

(iv) Not all aspects of TBU, e.g., healing flow [8], are captured by this model.